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State estimation for wave energy converters

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State estimation for wave energy converters

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1 Introduction

This report gives a brief discussion and examples on the topic of state estimation for wave energy converters (WECs). These methods are intended for use to enable real-time closed loop control of WECs. The algorithm for the optimal estimation of unknown inputs and state requires the system to be expressed using a discrete-time state-space model as [1]

$$x_{k+1} = Ax_k + Bu_k + Gd_k + w_k \tag{1a}$$

$$y_k = Cx_k + Du_k + Hd_k + v_k, \tag{1b}$$

where d_k is the unknown input at time step k (i.e. the excitation force $d_k = F_{e_k}$ or the excitation pressure $d_k = P_{e_k}$), u_k is the known input (i.e. the actuator's force $u_k = F_{a_k}$), y_k is the measurements vector (i.e. acceleration and pressure $y = [\ddot{z}, P]^T$), w_k and v_k are the process and measurements noise, respectively. Two examples are discussed here: using position and acceleration measurements (Section 2) and using pressure and acceleration measurements (Section 3).

2 Using position

This Section provides an example for using position and acceleration to predict the state of a WEC. Sample code for implementing this example is provided in Appendix A. The block diagram of the buoy's dynamic model is depicted in Fig. 1. The "intrinsic" model of the buoy G_i in continuous-time state space form is

$$\dot{x}_i = A_i x_i + B_i (F_e + F_a) \tag{2a}$$

$$y_i = C_i x_i + D_i (F_e + F_a) \tag{2b}$$

where the state vector is

$$x = \begin{bmatrix} z \\ \dot{z} \\ x_r \end{bmatrix}, \tag{3}$$

and

$$A_i = \begin{bmatrix} -\frac{B_f}{m+m_\infty} & -\frac{K}{m+m_\infty} & -\frac{C_r}{m+m_\infty} \\ 1 & 0 & \mathbf{0}_{1\times n_r} \\ B_r & \mathbf{0}_{n_r\times 1} & A_r \end{bmatrix} \qquad B_i = \begin{bmatrix} \frac{1}{m+m_\infty} \\ 0 \\ \mathbf{0}_{n_r\times 1} \end{bmatrix} \tag{4}$$

$$C_i = \begin{bmatrix} 0 & 1 & \mathbf{0}_{1\times n_r} \\ -\frac{B_f}{m+m_\infty} & -\frac{K}{m+m_\infty} & -\frac{C_r}{m+m_\infty} \end{bmatrix} \qquad D_i = \begin{bmatrix} 0 \\ \frac{1}{m+m_\infty} \end{bmatrix} \tag{5}$$

and where the matrices $A_r \in \mathbb{R}^{n_r \times n_r}$, $B_r \in \mathbb{R}^{n_r \times 1}$ and $C_r \in \mathbb{R}^{1 \times n_r}$ describe the radiation force F_r dynamics as

$$\dot{x}_r = A_r x_r + B_r \dot{z} \tag{6a}$$

$$F_r = C_r x_r. \tag{6b}$$

The mass of the buoy is denoted by m, the hydrostatic restoring coefficient by K, the friction coefficient by B_f and m_∞ is the asymptotic value of the added mass for $\omega \to \infty$. Two steps are now required to formulate system in (2) as required in (1)

1. Convert to discrete time
2. Derive matrices A, B, C, D, G and H

If the matrix A_i is not singular, then step 1 can be carried out by using the

$$A = e^{A_i T_c} \tag{7}$$

$$B = A_i^{-1} (A - I) B_i \tag{8}$$

where T_c is the sampling time. The matrices C, D, G and H are:

$$C = C_i \qquad D = D_i \tag{9}$$

$$G = B \qquad H = D. \tag{10}$$

The time-varying version of the algorithm given in 1 whereas the steady-state version (much faster computation) is given in 2.

Algorithm 1 Time-varying Unknown Input and State Estimator.

▷ Initialize:

$\hat{x}_{0|0} = \mathbb{E}[x_0]$

$\hat{d}_0 = H^{\dagger}\left(y_0 - C\hat{x}_{0|0} - Du_0\right)$

$P^x_{0|0} = \mathscr{P}^x_0$

$P^d_0 = \mathscr{P}^d_0$

$P^{xd}_0 = \mathscr{P}^{xd}_0$

$Q = \mathbb{E}[w\,w^T]$

$R = \mathbb{E}[v\,v^T]$

▷ Estimation loop for N time steps (Time step = T_c)

for $k = 1$ to N **do**

▷ One-Step prediction

$\hat{x}_{k|k-1} = A\,\hat{x}_{k-1|k-1} + B\,u_{k-1} + G\,\hat{d}_{k-1}$

$P^x_{k|k-1} = A\,P^x_{k-1|k-1}\,A^T + G\,P^{xd^T}_{k-1}\,A^T + A\,P^{xd}_{k-1}\,G^T + G\,P^d_{k-1}\,G^T + Q$

$\tilde{R}_k = C\,P^x_{k|k-1}\,C^T$

▷ Measurements update

$K_k = P^x_{k|k-1}\,C^T\,\tilde{R}_k^{-1}$

$L_k = K_k\left(I - H\left(H^T\,\tilde{R}_l^{-1}H\right)^{-1}H^T\,\tilde{R}_k^{-1}\right)$

$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k\left(y_k - C\,\hat{x}_{k|k-1} - D\,u_k\right)$

$P^x_{k|k} = (I - L_k\,C)\,P^x_{k|k-1}\,(I - L_k\,C)^T + L_k\,R\,L_k^T$

▷ Estimation of unknown input

$\tilde{R}^*_k = (I - CL_k)\,\tilde{R}_k\,(I - CL_k)^T$

$P^d_k = \left(H^T\tilde{R}^{*-1}_k H\right)^{-1}$

$M_k = P^d_k H^T\tilde{R}^{*-1}$

$\hat{d}_k = M_k\left(y_k - C\hat{x}_{k|k} - D\,u_k\right)$

$P^{xd}_k = -P^x_{k|k}C^T M_k^T + L_k R M_k^T$

end for

Algorithm 2 Steady-State Unknown Input and State Estimator.

▷ Initialize:

$\hat{x}_{0|0} = \mathbb{E}[x_0]$

$\hat{d}_0 = H^{\dagger}\left(y_0 - C\hat{x}_{0|0} - Du_0\right)$

$L_\infty = \lim_{k\to\infty} L_k$

$M_\infty = \lim_{k\to\infty} M_k$

▷ Estimation loop for N time steps (Time step = T_c)

for $k = 1$ to N **do**

$\hat{x}_{k|k-1} = A\,\hat{x}_{k-1|k-1} + B\,u_{k-1} + G\,\hat{d}_{k-1}$ ▷ One-Step prediction

$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_\infty\left(y_k - C\,\hat{x}_{k|k-1} - D\,u_k\right)$ ▷ State estimation

$\hat{d}_k = M_\infty\left(y_k - C\hat{x}_{k|k} - D\,u_k\right)$ ▷ Unknown input estimation

end for

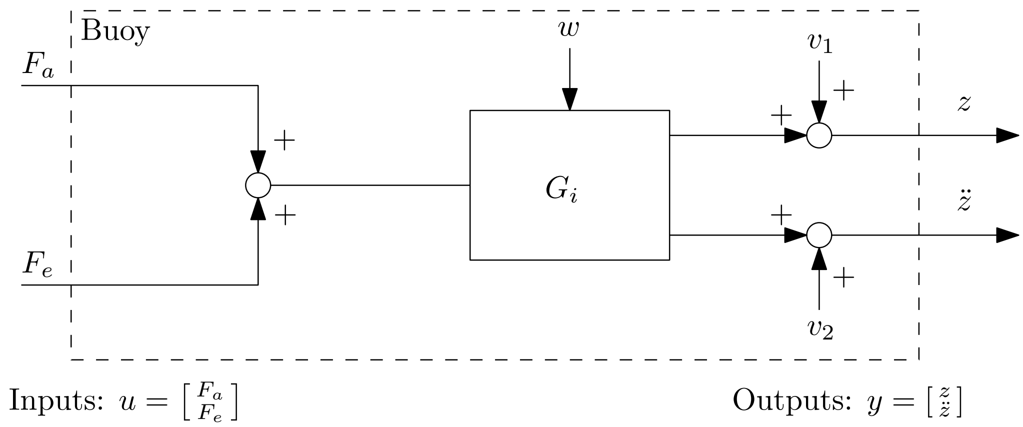

Figure 1. Block diagram of the buoy's dynamic model when position and acceleration are measured

3 Using pressure

This Section provides an example for using pressure and acceleration to predict the state of a WEC. Sample code for implementing this example is provided in Appendix B. The block diagram of the buoy's dynamic model is depicted in Fig. 1. The "intrinsic" model of the buoy G_i in continuous-time state space form is

$$\dot{x}_i = A_i x_i + B_i (F_e + F_a) \tag{11a}$$

$$y_i = C_i x_i + D_i (F_e + F_a) \tag{11b}$$

where the output vector y_i is

$$y_i = \begin{bmatrix} \ddot{z} \\ P_r \end{bmatrix}, \tag{12}$$

and the matrices composing the state space model in (11) have been identified form experimental data.

The state-space model of the excitation pressure in continuous-time (G_e) is:

$$\dot{x}_e = A_e x_e + B_e P_e \tag{13a}$$

$$F_e = C_e x_e + D_e P_e. \tag{13b}$$

According to the diagram in Fig. 2 the models in (11) and (13) can be combined to form the state-state space model

$$\dot{x} = A_c x + B_c \begin{bmatrix} F_a \\ P_e \end{bmatrix} \tag{14a}$$

$$y = C_c x + D_c \begin{bmatrix} F_a \\ P_e \end{bmatrix} \tag{14b}$$

where the state vector is

$$x = \begin{bmatrix} x_i \\ x_e \end{bmatrix} \tag{15}$$

and where the output vector is

$$y = \begin{bmatrix} \ddot{z} \\ P \end{bmatrix} = \begin{bmatrix} \ddot{z} \\ P_r + P_e \end{bmatrix} = \begin{bmatrix} \ddot{z} \\ P_r \end{bmatrix} + \begin{bmatrix} 0 \\ P_e \end{bmatrix} = y_i + \begin{bmatrix} 0 \\ P_e \end{bmatrix}. \tag{16}$$

The system matrices are

$$A_c = \begin{bmatrix} A_i & B_i C_e \\ \mathbf{0} & A_e \end{bmatrix} \qquad B_c = \begin{bmatrix} B_i & B_u D_e \\ \mathbf{0} & B_e \end{bmatrix} \tag{17}$$

$$C_c = \begin{bmatrix} C_i & D_i C_e \end{bmatrix} \qquad D_c = \begin{bmatrix} D_i & D_i D_e + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} \tag{18}$$

Two steps are now required to formulate system in (14) as required in (1):

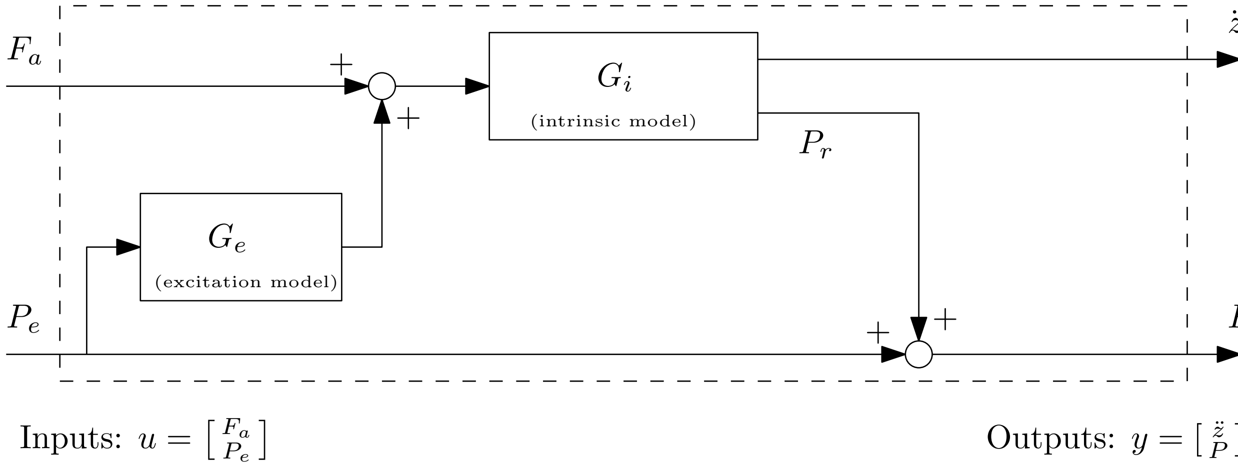

Figure 2. Block diagram of the buoy's dynamic model when pressure and acceleration are measured.

1. Convert to discrete time
2. Derive matrices A, B, C, D, G and H

If the matrix A_c is not singular, then step 1 can be carried out by using the

$$A = e^{A_c T_c} \tag{19}$$

$$\bar{B} = A_c^{-1}(A - I)B_c \tag{20}$$

where T_c is the sampling time. $\bar{B}$ and D_c are $2 \times n$ matrices: B is the first column of $\bar{B}$, G is the second column of $\bar{B}$, D is the first column of D_c and H is the second column of D_c, that is:

$$\bar{B} = [B \quad G] \tag{21}$$

$$C = C_c \qquad\qquad D_c = [D \quad H] \tag{22}$$

$$\tag{23}$$

The time-varying version of the algorithm given in 3 whereas the steady-state version (much faster computation) is given in 4.

Algorithm 3 Time-varying Unknown Input and State Estimator.

▷ Initialize:

$\hat{x}_{0|0} = \mathbb{E}[x_0]$

$\hat{d}_0 = H^{\dagger}\left(y_0 - C\hat{x}_{0|0} - Du_0\right)$

$P^x_{0|0} = \mathscr{P}^x_0$

$P^d_0 = \mathscr{P}^d_0$

$P^{xd}_0 = \mathscr{P}^{xd}_0$

$Q = \mathbb{E}[w w^T]$

$R = \mathbb{E}[v v^T]$

▷ Estimation loop for N time steps (Time step = T_c)

for $k = 1$ to N **do**

▷ One-Step prediction

$\quad \hat{x}_{k|k-1} = A\,\hat{x}_{k-1|k-1} + B\,u_{k-1} + G\,\hat{d}_{k-1}$

$\quad P^x_{k|k-1} = A\,P^x_{k-1|k-1}A^T + G\,P^{xd^T}_{k-1}A^T + A\,P^{xd}_{k-1}G^T + G\,P^d_{k-1}G^T + Q$

$\quad \tilde{R}_k = C\,P^x_{k|k-1}C^T$

▷ Measurements update

$\quad K_k = P^x_{k|k-1}C^T\,\tilde{R}_k^{-1}$

$\quad L_k = K_k\left(I - H\left(H^T\,\tilde{R}_l^{-1}H\right)^{-1}H^T\,\tilde{R}_k^{-1}\right)$

$\quad \hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k\left(y_k - C\,\hat{x}_{k|k-1} - D\,u_k\right)$

$\quad P^x_{k|k} = (I - L_k C)\,P^x_{k|k-1}\,(I - L_k C)^T + L_k R L_k^T$

▷ Estimation of unknown input

$\quad \tilde{R}^*_k = (I - CL_k)\,\tilde{R}_k\,(I - CL_k)^T$

$\quad P^d_k = \left(H^T\tilde{R}^{*-1}_k H\right)^{-1}$

$\quad M_k = P^d_k H^T\tilde{R}^{*-1}$

$\quad \hat{d}_k = M_k\left(y_k - C\hat{x}_{k|k} - D\,u_k\right)$

$\quad P^{xd}_k = -P^x_{k|k}C^T M_k^T + L_k R M_k^T$

end for

Algorithm 4 Steady-State Unknown Input and State Estimator.

▷ Initialize:

$\hat{x}_{0|0} = \mathbb{E}[x_0]$

$\hat{d}_0 = H^{\dagger}\left(y_0 - C\hat{x}_{0|0} - Du_0\right)$

$L_\infty = \lim_{k\to\infty} L_k$

$M_\infty = \lim_{k\to\infty} M_k$

▷ Estimation loop for N time steps (Time step = T_c)

for $k = 1$ to N **do**

$\quad \hat{x}_{k|k-1} = A\,\hat{x}_{k-1|k-1} + B\,u_{k-1} + G\,\hat{d}_{k-1}$ ▷ One-Step prediction

$\quad \hat{x}_{k|k} = \hat{x}_{k|k-1} + L_\infty\left(y_k - C\,\hat{x}_{k|k-1} - D\,u_k\right)$ ▷ State estimation

$\quad \hat{d}_k = M_\infty\left(y_k - C\hat{x}_{k|k} - D\,u_k\right)$ ▷ Unknown input estimation

end for

References

[1] Sze Zheng Yong, Minghui Zhu, and Emilio Frazzoli. A unified filter for simultaneous input and state estimation of linear discrete-time stochastic systems. *Automatica*, 63:321 – 329, 2016.

A Sample code: position and acceleration measurements

This section contain sample MATLAB code for implementing a position/acceleration state estimator for a WEC.

```
% this scripts is to test the Unified Linear Input and State Estimator
% (ULISE algorithm) described in
%
% S. Z. Yong, M. Zhu, and E. Frazzoli, A unified filter for simultaneous
% input and state estimation of linear discrete-time stochastic systems,
% Automatica, vol. 63, pp. 321 329, 2016.
%
% Both Time-Varying and Steady-State versions are implemented.
% Position and acceleration measurements are used to estimate state and
% excitation force
%
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% operated by National Technology and Engineering Solutions of Sandia,
% LLC., a wholly owned subsidiary of Honeywell International, Inc., for the
% U.S. Department of Energy?s National Nuclear Security Administration
% under contract DE-NA0003525.
%
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% 2017


clc
clear

% load identified parametric WEC model model
WEC = load('WEC_param_model_1DOF.mat');

Tc = 1e-3; % sampling time
N = 5e4; % number of simulation steps

% load excitation force and interploate
Fe = load('Exc_time_series.mat');
t = (1:N)*Tc;
d = interp1(Fe.t_trim, Fe.Fexc_td, t, 'pchip')*1e-3; %(excitation force in kN)

u = 0.5*sin(2*pi*0.75*t); % control input (PTO force in kN). Open loop, no control implemented

mass = 858.3987;
Ainf_hat = 822.3799;
K = 2.3981e+04;

Ar = WEC.rad_sys.a;
Br = WEC.rad_sys.b;
Cr = WEC.rad_sys.c;

Bf = WEC.B_eq_mat(9);

Ac = [-Bf/(mass + Ainf_hat)    -K/(mass + Ainf_hat)        -Cr/(mass + Ainf_hat);
       1                             0                         zeros(1,length(Ar));
       Br                        zeros(length(Ar),1)               Ar              ];

% input matrix (1e3 factor is used to acccount for the forces that expressed are in kN)
Bc = 1e3*[1/(mass + Ainf_hat);
          0                  ;
          zeros(length(Ar),1)];

% measurements: position and acceleration
```

```
C = [0 1 zeros(1,length(Ar));
       Ac(1,:)                ];

D = [0; Bc(1)];

n = size(C,2);
p = size(C,1);

% convert continuous time model to discrete time
A = expm(Ac*Tc);
B = Ac\(A - eye(n))*Bc;

G = B;
H = D;

% process noise
w = .00001*(rand(N,4)-0.5);
% measurements noise
v = (rand(N,p)-0.5) * diag([.0001,0.05]);

Q = cov(w);
R = cov(v);

%% Time varying filter

% initialize variables
P_x_k_k = 0.001*eye(n);
P_xd_k = 0.001*ones(n,1);
P_d_k = 0.001;

x = zeros(n,1);
x_k_k = x;
d_k = 0;

In = eye(n);
Ip = eye(p);

% preallocation
d_k_vec = zeros(N,1);
x_k_vec = zeros(n,N);
x_vec = zeros(n,N);
y_vec = zeros(p,N);


tic

for ii = 1:N

    x = A*x + B*u(ii) + G*d(ii) + w(ii,:)';
    y = C*x + D*u(ii) + H*d(ii) + v(ii,:)';

    x_vec(:,ii) = x;
    y_vec(:,ii) = y;

    x_k_k1 = A*x_k_k + B*u(ii) + G*d_k;
    P_x_k_k1 = A*P_x_k_k*A' + G*P_xd_k'*A' + A*P_xd_k*G' + G*P_d_k*G' + Q;
    R_t_k = C*P_x_k_k1*C' + R;
    Kk = P_x_k_k1*C'/R_t_k;
    Lk = Kk*(Ip-((H/((H'/R_t_k)*H))*H'/R_t_k ));
    x_k_k = x_k_k1 + Lk*(y - C*x_k_k1 - D*u(ii) );
    P_x_k_k = (In-Lk*C)*P_x_k_k1*(In-Lk*C)' + Lk*R*Lk';
    R_ts_k = (Ip-C*Lk)*R_t_k*(Ip-C*Lk)';
    P_d_k = inv((H'/R_ts_k)*H);
    Mk = ((H'/R_ts_k)*H)\(H'/R_ts_k);
    d_k = Mk*(y - C*x_k_k - D*u(ii));
    P_xd_k = -P_x_k_k*C'*Mk' + Lk*R*Mk';

    d_k_vec(ii) = d_k;
```

```
    x_k_vec(:,ii) = x_k_k;

end

t_tv = toc;
disp('done TV')

%% steady state filter

L_inf = Lk;
M_inf = Mk;

% preallocation
d_k_vec_inf = zeros(N,1);
x_vec_inf = zeros(n,N);
y_vec_inf = zeros(p,N);
x_vec_est_inf = x_vec_inf;

% initialization
x = zeros(n,1);
x_k_k_inf = zeros(n,1);
d_k1_inf = 0;

tic
for ii = 1:N

    x = A*x + B*u(ii) + G*d(ii) + w(ii,:)';
    y = C*x + D*u(ii) + H*d(ii) + v(ii,:)';

    x_vec_inf(:,ii) = x;
    y_vec_inf(:,ii) = y;

    x_k_k1_inf = A*x_k_k_inf + B*u(ii) + G*d_k1_inf;
    x_k_k_inf = x_k_k1_inf + L_inf*(y - C*x_k_k1_inf - D*u(ii) );
    d_k1_inf = M_inf*(y - C*x_k_k_inf - D*u(ii));

    d_k_vec_inf(ii) = d_k1_inf;
    x_vec_est_inf(:,ii) = x_k_k_inf;
end

t_ss = toc;
disp('done SS')


%% plotting

disp(['Time to compute Time-Varying filter: ' num2str(t_tv) 's'])
disp(['Time to compute Steady-State filter: ' num2str(t_ss) 's'])

figure(1)
plot(t, d_k_vec', t, d)
xlabel('time (s)')
ylabel('(kN)')
grid on
title('Time-Varying filter')
legend({'${\hat{F}e}_{\infty}$', '$Fe$'}, 'Interpreter', 'latex')

figure(2)
plot(t, d_k_vec_inf, t, d)
xlabel('time (s)')
ylabel('(kN)')
grid on
title('Steady-State filter')
legend({'$\hat{F}e$', '$Fe$'}, 'Interpreter', 'latex')

figure(3)
plot(t, d_k_vec - d_k_vec_inf)
grid on
```

```
xlabel('Time (s)')
title('Difference between Time-Varying and Steady-State filters')
legend('e_d')

figure(4)
subplot 211
plot(t, y_vec(1,:)')
xlabel('Time (s)')
ylabel('(m)')
title('Measured (noisy) Outputs')
legend({'$z$'}, 'Interpreter', 'latex')
grid on
subplot 212
plot(t, y_vec(2,:)')
xlabel('Time (s)')
ylabel('(m/s^2)')
legend({'$\ddot{z}$'}, 'Interpreter', 'latex')
grid on


figure(5)
subplot 211
plot(t, x_vec(1,:)', t, x_k_vec(1,:)', t, x_vec_est_inf(1,:)')
grid on
xlabel('time (s)')
ylabel('(m/s)')
legend({'$v$', '$\hat{v}$', '$\hat{v}_{\infty}$'}, 'Interpreter', 'latex')
title('Estimated states')
grid on

subplot 212
plot(t, x_vec(2,:)', t, x_k_vec(2,:)', t, x_vec_est_inf(2,:)')
grid on
xlabel('time (s)')
ylabel('(m)')
legend({'$z$', '$\hat{z}$', '$\hat{z}_{\infty}$'}, 'Interpreter', 'latex')
grid on
```

State_and_unknown_input_estimator_position_acceleration.m

B Sample code: pressure and acceleration measurements

This section contain sample MATLAB code for implementing a pressure/acceleration state estimator for a WEC.

```
% this scripts is to test the Unified Linear Input and State Estimator
% (ULISE algorithm) described in
%
% S. Z. Yong, M. Zhu, and E. Frazzoli, A unified filter for simultaneous
% input and state estimation of linear discrete-time stochastic systems,
% Automatica, vol. 63, pp. 321 329 , 2016.
%
% Both Time-Varying and Steady-State versions are implemented. Pressure and
% acceleration measurements are used to estimate state and excitation force
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% LLC., a wholly owned subsidiary of Honeywell International, Inc., for the
% U.S. Department of Energy?s National Nuclear Security Administration
% under contract DE-NA0003525.
%
% G. Bacelli, R. Coe
% Sandia National Laboratories
% 2017


clc
clear

% load identified parametric WEC model model
WEC = load('WEC_param_model_1DOF.mat'); % radiatiojn impedance model
Gr = struct2array(load('Gr_model.mat')); % radiation pressure model
Ge = struct2array(load('Ge_model.mat')); % excitation pressure model

Tc = 1e-3;
N = 5e4;

% load excitation force
Fe = load('Exc_time_series.mat'); %(excitation force in kN)
t = (1:N)*Tc;
d = interp1(Fe.t_trim, Fe.Fexc_td, t, 'pchip')*1e-3; % control input (PTO force in kN). Open loop
    , no control implemented

u = .5*sin(2*pi*0.75*t);

mass = 858.3987;
Ainf_hat = 822.3799;
K = 2.3981e+04;

Ar = WEC.rad_sys.a;
Br = WEC.rad_sys.b;
Cr = WEC.rad_sys.c;

Bf = WEC.B_eq_mat(9);

Am = [-Bf/(mass + Ainf_hat)    -K/(mass + Ainf_hat)        -Cr/(mass + Ainf_hat);
    1                  0                      zeros(1,length(Ar));
    Br                 zeros(length(Ar),1)    Ar];

% input matrix (1e3 factor is used to acccount for the forces that expressed are in kN)
Bm = 1e3*[1/(mass + Ainf_hat);
    0                   ;
    zeros(length(Ar),1)  ];
```

```
Cm = Am(1,:);

Dm = Bm(1);

Ai = blkdiag(Am, Gr.A);
Bi = [Bm; Gr.B];
Ci = blkdiag(Cm, Gr.C);
Di = [Dm; Gr.D];

ni = size(Ai,1);
ne = size(Ge.A,1);
Ac = [Ai,            Bi*Ge.C;
      zeros(ne, ni) Ge.A];

Bc = [Bi, Bi*Ge.D; zeros(ne,1), Ge.B];

C = [Ci, Di*Ge.C];
D = [Di, Di*Ge.D + [0;1] ];

sys_c = ss(Ac, Bc, C, D);
sys_d = c2d(sys_c, Tc);

Ge_d = c2d(Ge,Tc);

Ce = Ge_d.C;
De = Ge_d.D;

n = size(C,2);
p = size(C,1);

% convert continuous time model to discrete time
A = expm(Ac*Tc);
B = Ac\(A - eye(n))*Bc;

G = B(:,1);
B = B(:,1);

H = D(:,2);
D = D(:,1);

% process noise
w = .00001*(rand(N,ni+ne)-0.5);
% measurements noise
v = (rand(N,p)-0.5) * diag([.05, 0.05]);

Q = cov(w);
R = cov(v);

%% Time Varying filter

% initialize variables
P_x_k_k = 0.001*eye(n);
P_xd_k = 0.001*ones(n,1);
P_d_k = 0.001;

x = zeros(n,1);
x_k_k = x;
d_k = 0;

In = eye(n);
Ip = eye(p);

% preallocation
x_vec = zeros(n,N);
y_vec = zeros(p,N);
d_k_vec = zeros(N,1);
x_k_vec = zeros(n,N);
```

```
Fe_est = zeros(N,1);
Fe = Fe_est;

tic

for ii = 1:N

    x = A*x + B*u(ii) + G*d(ii) + w(ii,:)';
    y = C*x + D*u(ii) + H*d(ii) + v(ii,:)';

    Fe(ii) = Ce*x(ni+1:end) + De*d(ii);

    x_vec(:,ii) = x;
    y_vec(:,ii) = y;

    x_k_k1 = A*x_k_k + B*u(ii) + G*d_k;
    P_x_k_k1 = A*P_x_k_k*A' + G*P_xd_k'*A' + A*P_xd_k*G' + G*P_d_k*G' + Q;
    R_t_k = C*P_x_k_k1*C' + R;
    Kk = P_x_k_k1*C'/R_t_k;
    Lk = Kk*(Ip-((H/((H'/R_t_k)*H))*H'/R_t_k ));
    x_k_k = x_k_k1 + Lk*(y - C*x_k_k1 - D*u(ii) );
    P_x_k_k = (In-Lk*C)*P_x_k_k1*(In-Lk*C)' + Lk*R*Lk';
    R_ts_k = (Ip-C*Lk)*R_t_k*(Ip-C*Lk)';
    P_d_k = inv((H'/R_ts_k)*H);
    Mk = ((H'/R_ts_k)*H)\(H'/R_ts_k);
    d_k = Mk*(y - C*x_k_k - D*u(ii));
    P_xd_k = -P_x_k_k*C'*Mk' + Lk*R*Mk';

    d_k_vec(ii) = d_k;
    x_k_vec(:,ii) = x_k_k;
    Fe_est(ii) = Ce*x_k_k(ni+1:end) + De*d_k;

end

t_tv = toc;
disp('done TV')

%% Steady State filter

L_inf = Lk;
M_inf = Mk;

% preallocation
d_k_vec_inf = zeros(N,1);
x_vec_inf = zeros(n,N);
y_vec_inf = zeros(p,N);
x_vec_est_inf = x_vec_inf;
Fe_est_inf = zeros(N,1);

% initialization
x = zeros(n,1);
x_k_k_inf = zeros(n,1);
d_k1_inf = 0;

tic
for ii = 1:N

    x = A*x + B*u(ii) + G*d(ii) + w(ii,:)';
    y = C*x + D*u(ii) + H*d(ii) + v(ii,:)';

    x_vec_inf(:,ii) = x;
    y_vec_inf(:,ii) = y;

    x_k_k1_inf = A*x_k_k_inf + B*u(ii) + G*d_k1_inf;
    x_k_k_inf = x_k_k1_inf + L_inf*(y - C*x_k_k1_inf - D*u(ii) );
    d_k1_inf = M_inf*(y - C*x_k_k_inf - D*u(ii));

    d_k_vec_inf(ii) = d_k1_inf;
```

```
    x_vec_est_inf(:,ii) = x_k_k_inf;
    Fe_est_inf(ii) = Ce*x_k_k_inf(ni+1:end) + De*d_k1_inf;
end

t_ss = toc;
disp('done SS')

%% plotting

disp(['Time to compute Time-Varying filter: ' num2str(t_tv) 's'])
disp(['Time to compute Steady-State filter: ' num2str(t_ss) 's'])


figure(1)
subplot 211
plot(t, d_k_vec', t, d)
xlabel('time (s)')
ylabel('(kPa)')
grid on
legend({'$\hat{P}e$', '$Pe$' }, 'Interpreter', 'latex')
title('Time-Varying filter: Unknown input (Excitation pressure Pe)')

subplot 212
plot(t, Fe_est, t, Fe)
grid on
ylabel('(kN)')
xlabel('time (s)')
title('Excitation force')
legend({'$\hat{F}e$', '$Fe$'}, 'Interpreter', 'latex')

figure(2)
subplot 211
plot(t, d_k_vec_inf', t, d)
xlabel('time (s)')
ylabel('(kPa)')
grid on
legend({'$\hat{P}e$', '$Pe$' }, 'Interpreter', 'latex')
title('Steady-State filter: Unknown input (Excitation pressure Pe)')

subplot 212
plot(t, Fe_est_inf, t, Fe)
grid on
ylabel('(kN)')
xlabel('time (s)')
title('Excitation force')
legend({'$\hat{F}e$', '$Fe$'}, 'Interpreter', 'latex')

figure(3)
plot(t, d_k_vec - d_k_vec_inf)
grid on
xlabel('Time (s)')
title('Difference between Time-Varying and Steady-State filters')
legend('e_d')

figure(4)
subplot 211
plot(t, y_vec(1,:)')
xlabel('Time (s)')
ylabel('(m/s^2)')
title('Measured (noisy) Outputs')
legend({'$\ddot{z}$'}, 'Interpreter', 'latex')
grid on
subplot 212
plot(t, y_vec(2,:)')
xlabel('Time (s)')
ylabel('(kPa)', 'Interpreter', 'latex')
grid on
legend({'$P$'}, 'Interpreter', 'latex')
```

```
figure(5)
subplot 211
plot(t, x_vec(1,:)', t, x_k_vec(1,:)', t, x_vec_est_inf(1,:)')
grid on
xlabel('time (s)')
ylabel('(m/s)')
legend({'$v$', '$\hat{v}$', '$\hat{v}_{\infty}$'}, 'Interpreter', 'latex')
title('Estimated states')
grid on

subplot 212
plot(t, x_vec(2,:)', t, x_k_vec(2,:)', t, x_vec_est_inf(2,:)')
grid on
xlabel('time (s)')
ylabel('(m)')
legend({'$z$', '$\hat{z}$', '$\hat{z}_{\infty}$'}, 'Interpreter', 'latex')
grid on
```

State_and_unknown_input_estimator_pressure_acceleration.m

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