

Wen-Chi Cheng, PhD
Nuclear Computational Scientists (INL)

Gang Li, PhD
Assistant Professor (MSU)

Mucun Sun, PhD
Power Systems Engineer (INL)

Probabilistic Physics-of-Failure based Reliability Assessment for an Infinitely Variable Transmission

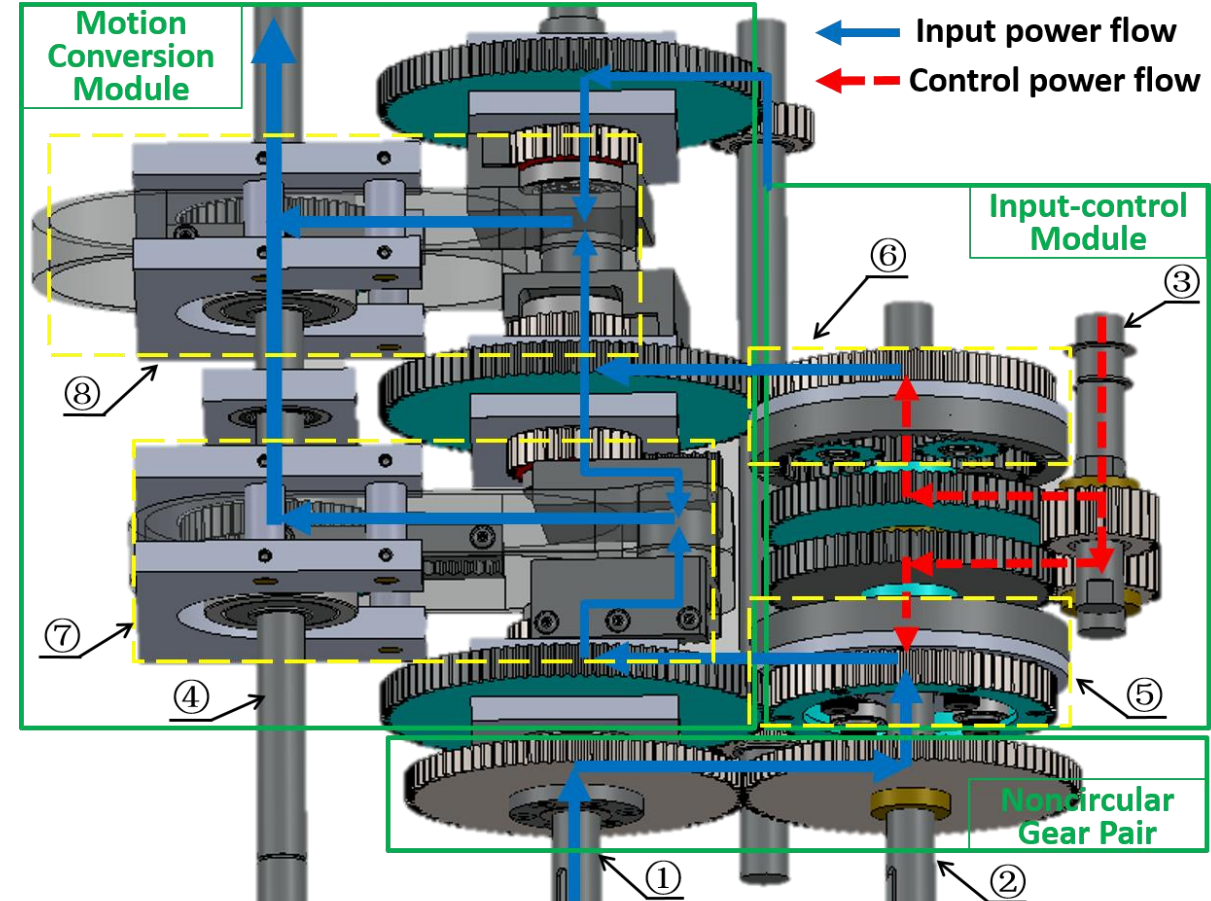
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Research Objective and Tasks

- **Objective:** Perform the following tasks to support New Technology Qualification (NTQ) process for infinitely variable transmission (IVT) design in tidal energy converter.
 - Task 1: Failure modes and effects analysis (FMEA) on the gear box
 - both the root bending fatigue in non-circular gear pair and planet gear sets were identified as the dominant failure mechanisms (WE336)
 - Task 2: Probabilistic Physics-of-Failure (PPOF) based reliability assessment-stress-strength analysis (WE206)

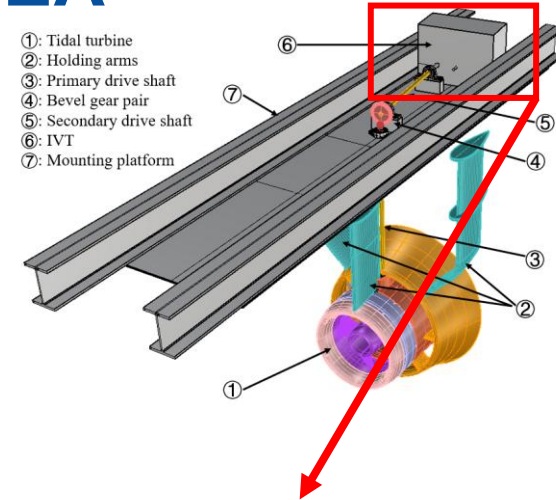
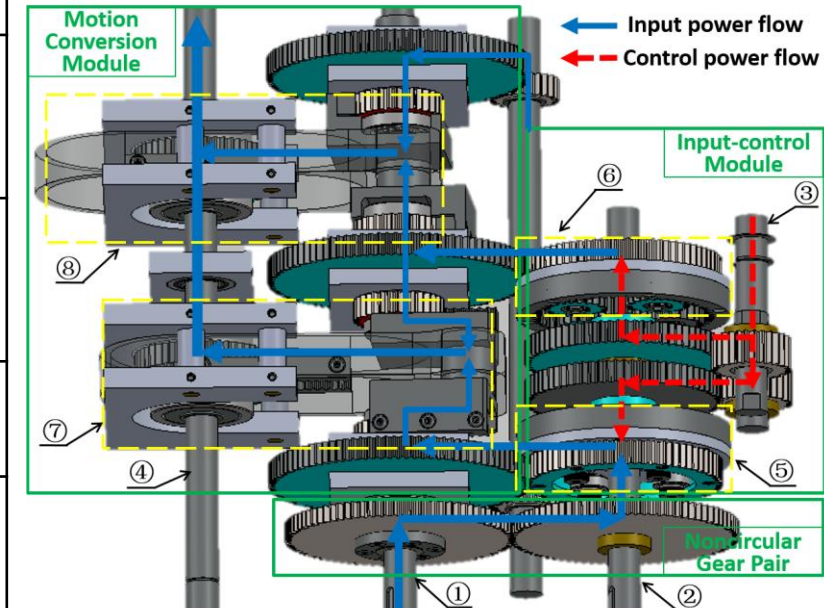


Research Overview

- Pick the most dominant failure modes from FMEA (loss of torque transmission)
 - Stress overload (failure mechanism-1)
 - Tooth bending fatigue (failure mechanism-2)
- Perform stress/load estimation for gear box as a function of tide current velocity
- Evaluate the strength of the gear for multiple materials
- Quantitative analysis for stress-strength analysis (for overloaded conditions)
- Quantitative analysis for high-cycle fatigue analysis (low stress condition)
- Reliability analysis by combining the over-loaded and high-cycle fatigue analysis

2.1 Pick the most dominant failure modes from FMEA

IV Gear Systems	Category	Failure Mode	Failure Mechanism (Root Causes)	RPN (Severity*Frequency*Detection)
Noncircular gear pair	A.1	Loss of torque Transmission	Root bending fatigue	320 (10*4*8)
Planetary gear sets	B.3	Loss of torque Transmission	Excessive vibration/noise; Overheating; Locked rotation	320 (10*4*8)
Output gears	D.1	Loss of output torque contribution	Root bending fatigue of output gear Teeth	240 (10*3*8)
Planetary gear sets	B.1	Loss of combined speed accuracy	Sun gear tooth deflection under load	216 (9*3*8)
Rack–pinion sets	C.1	Excess backlash	Abrasive wear of rack and pinion teeth	210 (6*7*5)
Noncircular gear pair	A.3	Loss of modulation accuracy	Fretting wear at hub-shaft interface	200 (8*5*5)



The challenge: Subsea Energy Harvesting

Why is this difficult?



Harsh Environment

Subsea deployment with variable tidal currents



Limited Access

High maintenance costs, difficult repairs



Variable Loads

Fluctuating tidal speeds create cyclic stress



No Field Data

New technology with limited reliability history

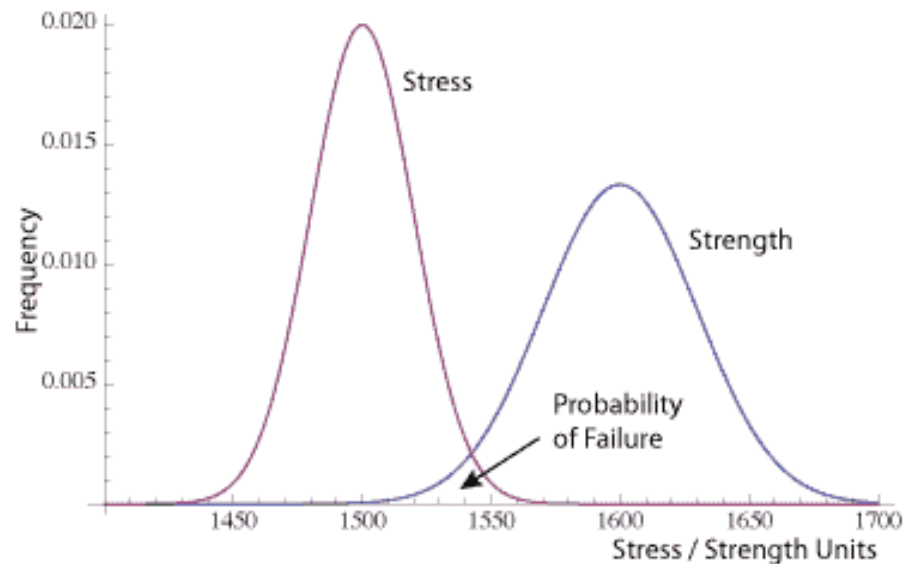
Key Question: How reliable is the IVT, and how long will it last underwater?



The Probabilistic-Physics of Failure (PPoF) Approach is proposed

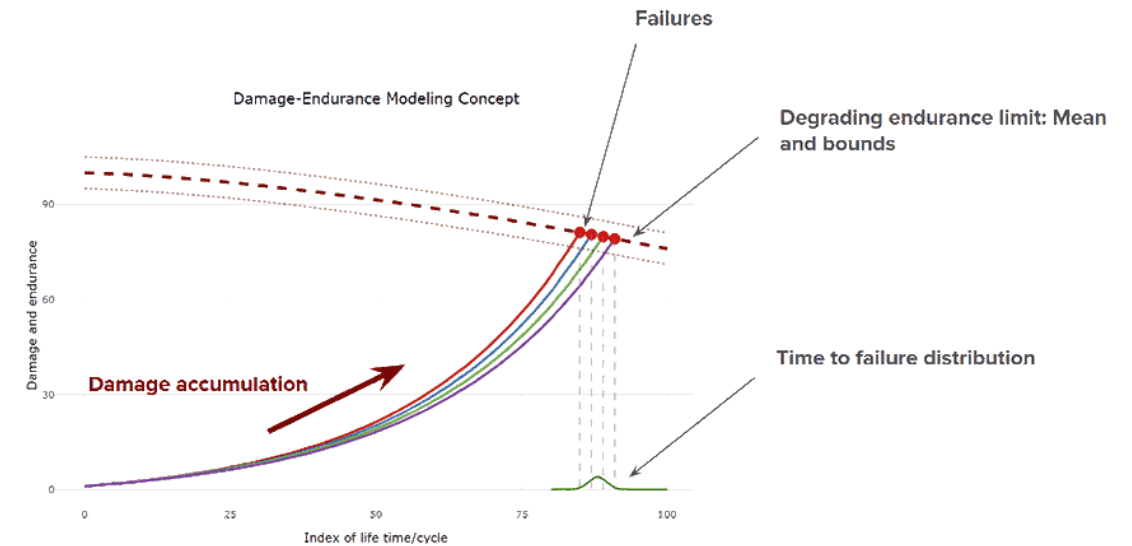
Probabilistic Physics-of-Failure Methodology

Stress-strength Analysis



Good for overloaded condition analysis

Damage-Endurance



Good for wear, corrosion, erosion, and fatigue analysis

M. Modarres, M. P. Kaminskiy, and V. Krivtsov, *Reliability engineering and risk analysis: a practical guide*. CRC press, 2016

Stress Characterization (Inputs and model)

$$\sigma_b = \frac{T_{in}(t)}{bmYr_{NG2}(t)}$$

$$r_{NG2}(t) = \frac{D}{1 + i_n(\theta(t))}$$

$$i_n(\theta(t)) = \frac{\pi}{2\sqrt{2}} \max \left[\sin\theta_n, \sin\left(\theta_n - \frac{\pi}{2}\right), \sin(\theta_n - \pi), \sin\left(\theta_n - \frac{3\pi}{2}\right) \right],$$

$$\theta_n(t = 0) = 0; \theta_n(t + 1) = \theta_n(t) + \frac{2\pi N_p(t)}{60 i_n(\theta(t))},$$

$$T_{in}(t) = \frac{60P_t}{2\pi N_p(t)},$$

[1] G. Li and W. Zhu, "Control and flume flow experiments of a tidal current energy converter with an infinitely variable transmission," IEEE/ASME Transactions on Mechatronics, vol. 29, no. 4, pp. 2803-2811,(2024).

[2] G. Li and W. Zhu, "Design and power loss evaluation of a noncircular gear pair for an infinitely variable transmission," Mechanism and Machine Theory, vol. 156, p. 104137,(2021).

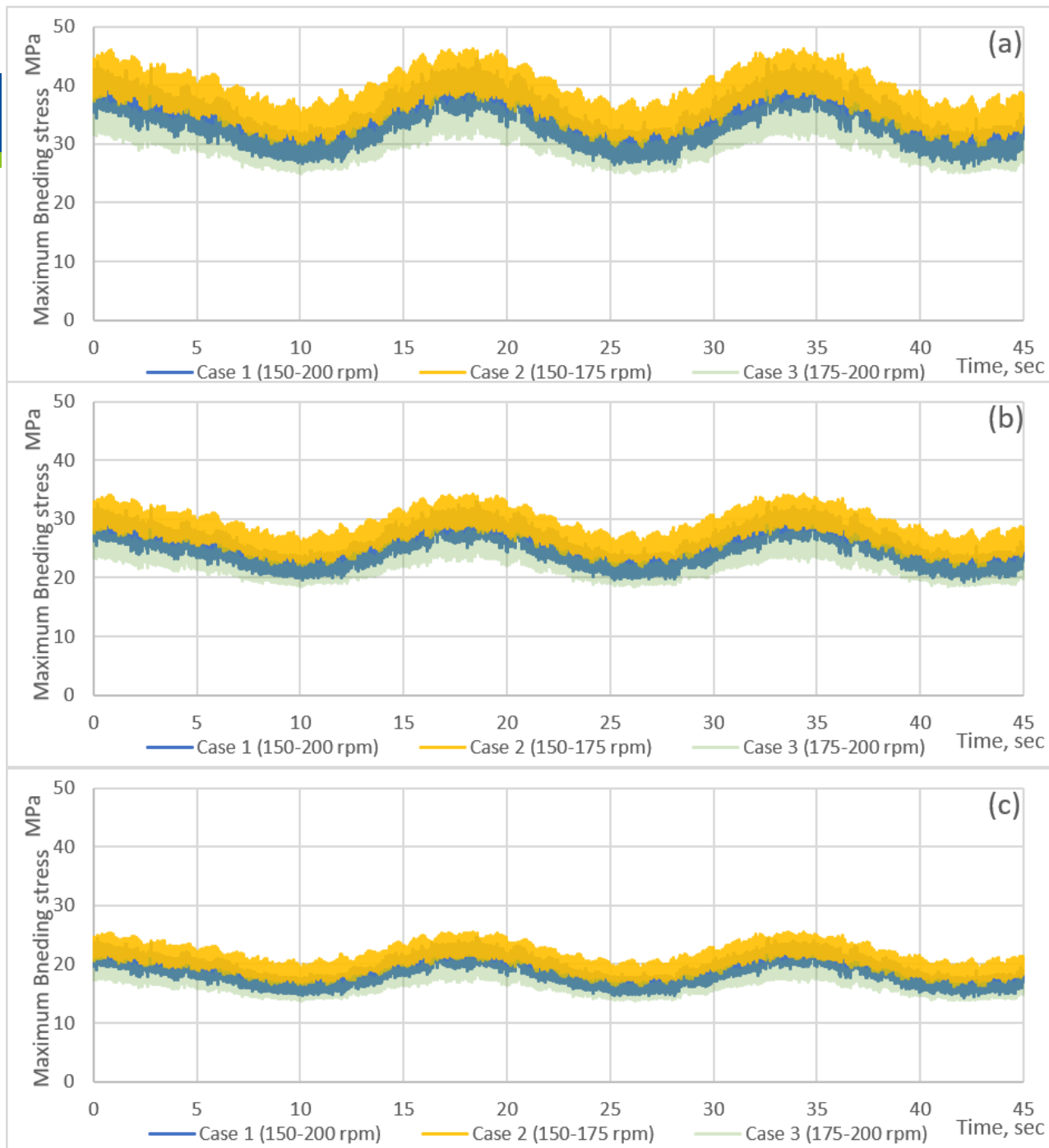
[7] RoyMech. "Spur Gears: Design, Formula, Terminology and Calculations." https://www.roytech.co.uk/Useful_Tables/Drive/Gears.html (accessed May 21, 2026).

*Assume that it is applicable for 42-tooth with 20 degrees of pressure angle

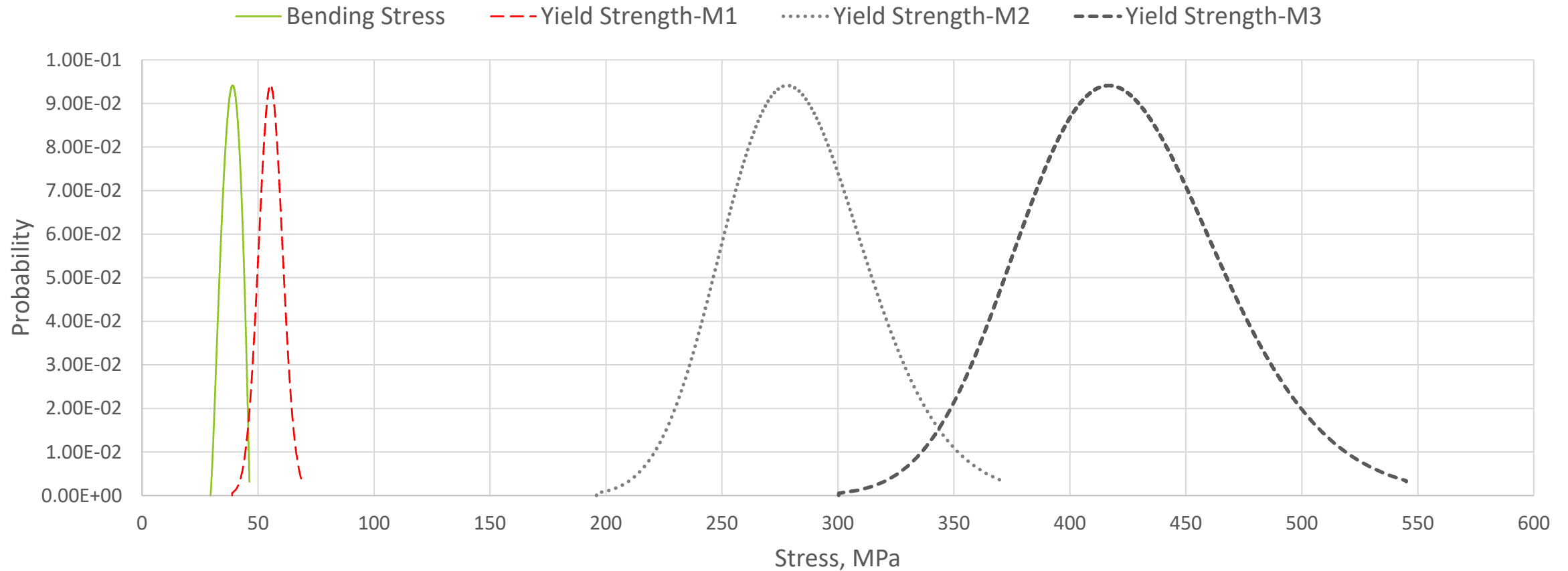
Parameters	Notations	Values	Units
Face width	b	0.02 [2]	m
Modules	m	0.003 [2]	m
Lewis form factor	Y	0.394 [7]*	Unitless
Center distance	D	0.135 [2]	m
Transmitted power from the turbine	P_t	750 [1]	W
Pitch radius for NG2-Case 1	$r_{NG2}(\varphi(t))$	0.064 to 0.076 [2]	m
Pitch radius for NG2-Case 2	$r_{NG2}(\varphi(t))$	0.064 to 0.076 [2]	m
Pitch radius for NG2-Case 3	$r_{NG2}(\varphi(t))$	0.064 to 0.076 [2]	m
Input rotational speed (150-200 rpm)-Case 1	$N_{p,1}(t)$	142 to 214 [1]	rpm
Input rotational speed (150-175 rpm)-Case 2	$N_{p,2}(t)$	137 to 186 [1]	rpm
Input rotational speed (175-200 rpm)-Case 3	$N_{p,3}(t)$	164 to 225 [1]	rpm

Stress Characterization (Results)

- All bending stresses are all less than 50 MPa (**limited concern if material is selected properly**)
- The stress ranges between the maximum and minimum stresses are less than 17 MPa (**high cycle fatigue problem**)

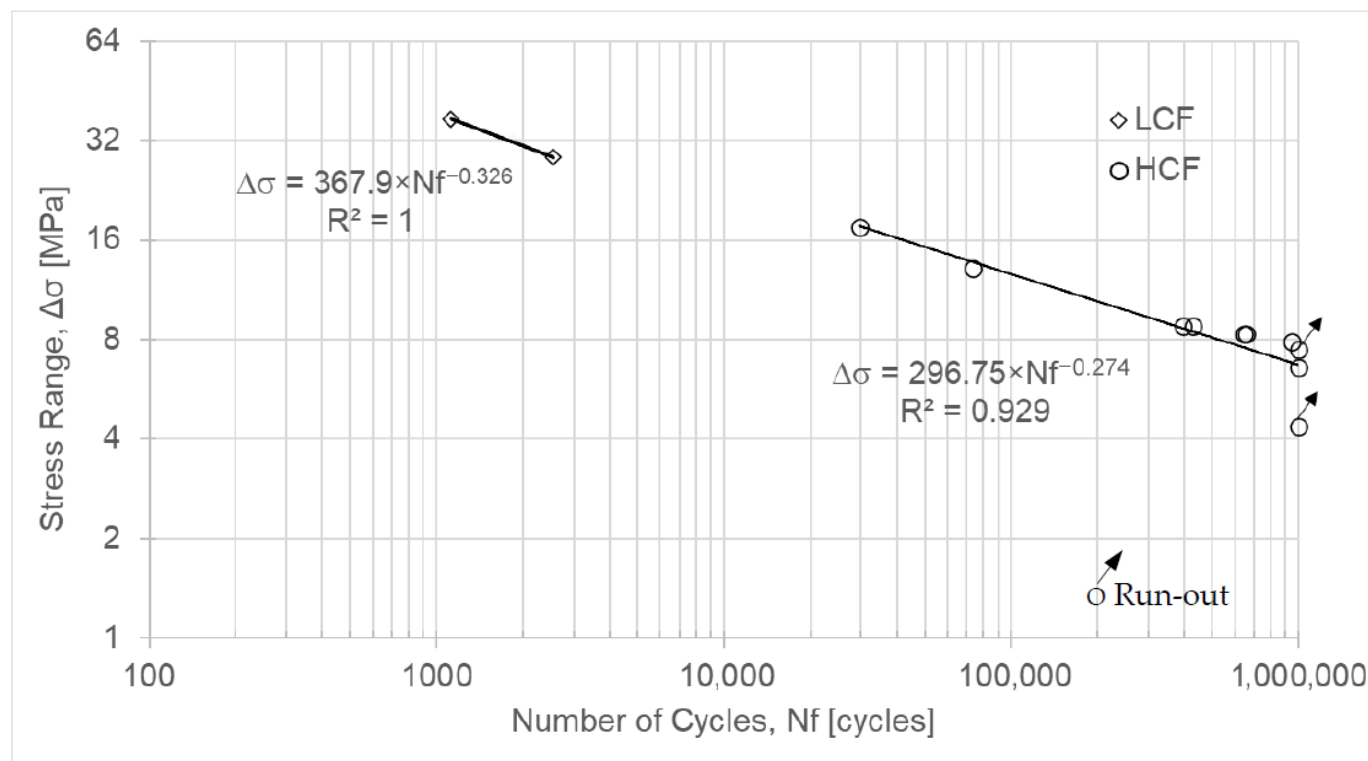


Phase I: Stress-Strength Analysis



Types of Material	Yield strength (MPa)
M1: Nylon-12	Normal ($\mu=55$, $\sigma=4$)
M2: 6061-T6/T651 aluminum	Lognormal (mean=276, std=23.6)
M3: AISI 4140 low-alloy steel	Lognormal (mean=415, std=33)

Phase II: Damage-endurance (S-N curves)



Pressure Angle	10 degrees	20 degrees	30 degrees
Maximum stress: σ_{max} , MPa	46.3	34.3	25.5
Minimum stress: σ_{min} , MPa	29.5	21.8	16.3
Period of the cycle: t_f , sec	17.5	17.5	17.5
Lifetime: t_{life} , year	7.2	21.5	62.7
Failure rate, λ (1/year)	0.14	0.05	0.02
Reliability (one year), $Re_I(t = 1) = e^{-\lambda t}$	86.94%	95.12%	98.02%

Â. Rodrigues, R. Branco, M. Franco, R. Silva, C. Malça, and R. F. Martins, "Mechanical, Fatigue, and Thermal Characterization of ASA, Nylon 12, PC, and PC-ABS Manufactured by Fused Filament Fabrication (FFF)," *Polymers*, vol. 18, no. 2, p. 302,(2026).

Combined Reliability Estimation

$$\text{Total Reliability} = 1 - (P_{\text{overload}} + P_{\text{fatigue}} - P_{\text{overload}} \times P_{\text{fatigue}})$$

10° Pressure Angle

Phase I (overload)

0.47%

Phase II (fatigue)

13.1%

Combined

86.5%

20° Pressure Angle

Phase I (overload)

<0.0001%

Phase II (fatigue)

4.88%

Combined

95.1%

30° Pressure Angle

Phase I (overload)

<0.0001%

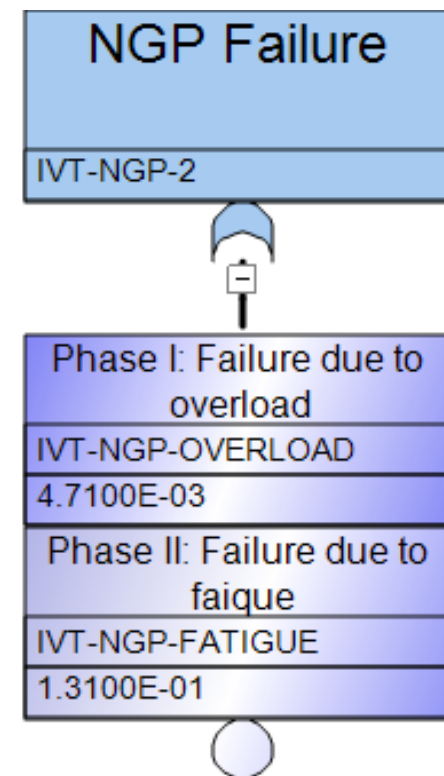
Phase II (fatigue)

1.98%

Combined

98.0%

Key Takeaway: Fatigue dominates for all angles. Phase I only matters at 10° with Nylon-12.



Conclusions and Future works

- Failure Mode and Effect Analysis (FMEA) indicates that tooth bending fatigue is a critical issue for non-circular gear pair in infinitely variable transmission (IVT) of tidal current energy converters for marine energy applications.
- Both stress-strength and damage-endurance from probabilistic physics-of-failure analysis was performed and concluded that damage-endurance analysis contributes more in the reliability assessment.
- Reliability ranges from 86% to 98% depending on the design pressure angle if a Nylon-12 is used.
- For the real tidal energy environment, it is expected that the power conversion is larger, leading to larger stress variation, which needs validation and verification using real testing data. (Paper #3)

Research Team

- Wen-Chi Cheng, PhD, wenchi.cheng@inl.gov
- Mucun Sun, PhD, Mucun.Sun@inl.gov
- Fazlur Rahman Bin Karim, PhD student, fazlurrahmanbin.karim@inl.gov
- Gang Li, Assistant professor at MSU, gli@me.msstate.edu
- ABS (American Bureau of Shipping)



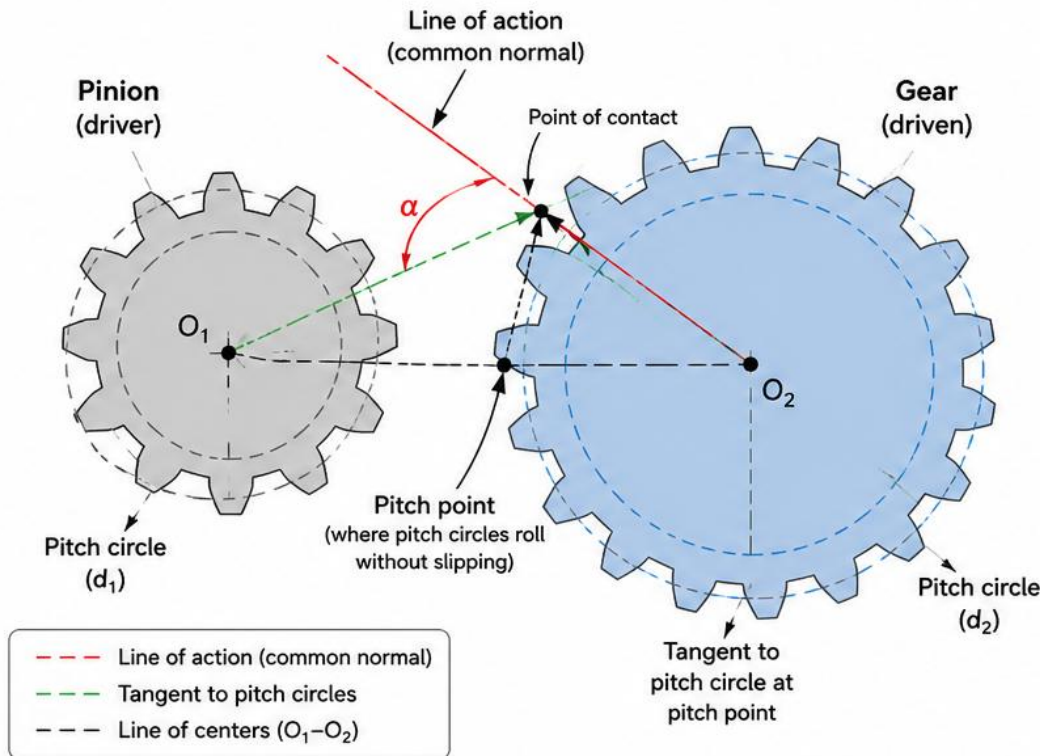
Idaho National Laboratory

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PRESSURE ANGLE (α) OF A GEAR PAIR

15

Definition: Pressure angle (α) is the angle between the line of action (the common normal at the point of tooth contact) and the tangent to the pitch circle at the pitch point.



KEY POINTS

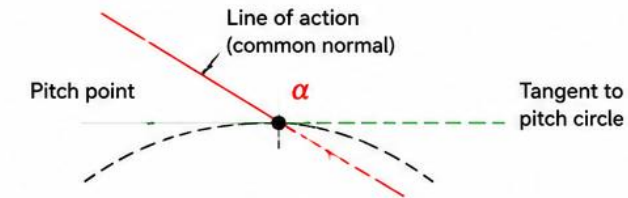
- **Pressure angle** is a property of the gear design (20° is most common).
- It affects load carrying capacity, smoothness, and axial thrust.
- Standard full-depth involute gears are specified by their pressure angle.

TYPICAL STANDARD PRESSURE ANGLES

Pressure Angle (α)	Common Use
14.5°	Older or special applications
20°	Most common (general purpose)
25°	Heavy load, high strength
30°	High load, reduced undercut

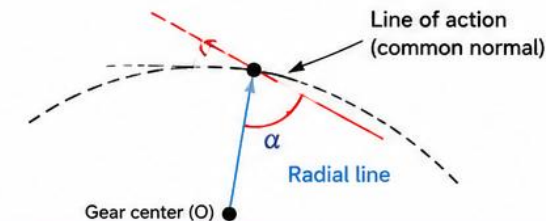
1) Common Definition (Most Used)

α = angle between the line of action and the tangent to the pitch circle at the pitch point.



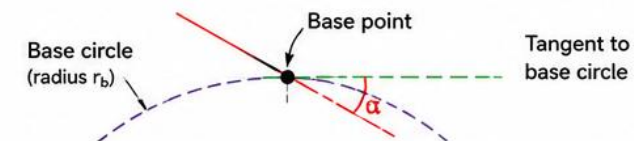
2) Equivalent Definition

α = angle between the line of action and the radial line (from the gear center to the point of contact) at the pitch point.



3) On the Base Circle (Used in Tooth Profile Geometry)

α = angle between the line of action and the tangent to the base circle at the base point.



WHAT DOES THE LINE OF ACTION REPRESENT?

The line of action is the direction of the resulting force transmitted between the teeth. The force acts along this line during meshing.

