

Probabilistic Physics-of-Failure based Reliability Assessment for an Infinitely Variable Transmission

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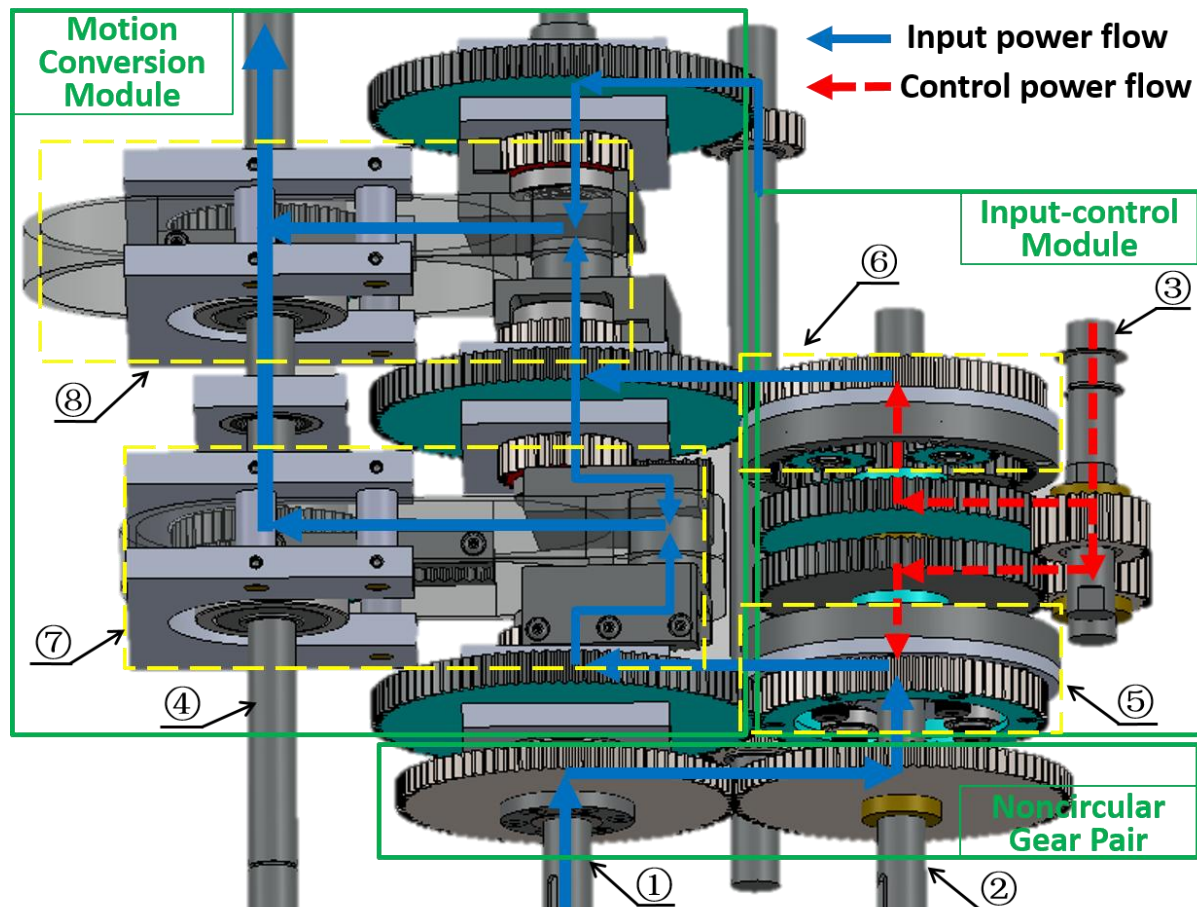
Abstract: Subsea tidal current energy converters (TCECs) face distinctive reliability challenges due to limited accessibility, high operational and maintenance costs, and highly variable operating loads that can undermine conventional deterministic design margins. This paper presents a Probabilistic Physics-of-Failure (PPoF) methodology for reliability assessment of an infinitely variable transmission (IVT) in TCEC applications to support risk-informed design and operational planning under high-torque and low-speed conditions. The PPoF methodology combines failure mode and effect analysis-driven screening to identify dominant failure modes and mechanisms with an environment-to-stress mapping that propagates lab-based testing tidal current velocity through turbine dynamics and drivetrain kinematics to obtain gear-tooth contact forces and tooth contact stress distributions, including time- and angle-dependent effects relevant to kinematically modulated gear pairs. A physics-based model is applied to evaluate tooth root bending stress and corresponding yield strength of the material, enabling a probabilistic stress–strength assessment of failure probability across operating and environmental conditions. The main contribution is the coupling of tidal-current-induced load modeling with standards-based mechanical strength analysis within a PPoF methodology, enabling uncertainty characterization and propagation with sparse test or field evidence. While PPoF methodology is used to demonstrate the IVTs in TCECs, it can be generalizable to other marine energy drivetrains where harsh environments, limited access, and operational variability challenge traditional reliability methods.

1. INTRODUCTION

The infinitely variable transmission (IVT) was designed in subsea tidal current energy converters (TCECs) for stabilizing the power outputs [1, 2]. The authors' previous research on the reliability of IVT showed that the loss of torque transmission is one of the major failure modes based on the failure mode and effect analysis (FMEA). From FMEA, both the root bending fatigue in non-circular gear pair and planet gear sets were identified as the dominant failure mechanisms. The real problem for the root bending failure is that a gear tooth behaves like a tiny cantilever beam. When mesh force acts on the tooth, the highest bending stress occurs near the tooth root fillet. If that bending stress keeps cycling up and down, a crack can initiate at the root and then grow over many cycles. Once that crack grows enough, the tooth can lose stiffness or break, which then shows up at system level as loss of torque transmission, more vibration, and possibly shutdown of IVT. The non-circular gear pair is the first module that takes power from the turbine directly which can be impacted by the velocity variations from the subsea environment while the planet gear sets are served as the main components in the input-control module as shown in

Figure 1. Both non-circular gear pair and planet gear sets should share the same torque. However, due to the non-circular shape of the gear for non-circular gear pair, there are additional stress variations coming from the pitch radius changes during the gear rotations in the non-circular gear pair; therefore, the non-circular gear pair is selected as the targeted component in this research.

Figure 1. Schematic drawing of the IVT. where 1 is the input shaft, 2 is the secondary shaft, 3 is the control shaft, 4 is the output shaft, 5 is the first planetary gear set (PGS), 6 is the second PGS, 7 is the first scotch-yoke system (SYS), and 8 is the second SYS. [1]



The authors' previous research on the reliability of IVT also developed a reliability and availability model for estimating the availability of IVT. As part of the required inputs, there is a need to estimate the failure rate for non-circular gear pairs. There are limited studies in the literature that focus on the root bending fatigue for non-circular gear pair; however, some of the relevant studies in this area that offer the key insights for this research. For example, Lin et al. [3] investigated the bending fatigue life for the curve faced gears which were different than the non-circular gear sets. The contact forces between the gears were simulated by multi-body dynamics model which serves as the inputs to the finite element analysis (FEA) to estimate stress. The fatigue life for the curved gears is calculated deterministically by varying different load values based on the S-N curve which is suitable for the gear made from a 40Cr material. While the physical geometry and the load conditions will be different in IVT of the TCEC applications. The general S-N curve for this specific material could be applicable, and the uncertainties of the S-N curve should be characterized if enough experimental data is available. Li and Wu [4] optimized the strength of the tooth based on asymmetric pressure angles that were proved to be sensitive to the maximum allowable strength of the material used to manufacture the non-circular gear pair. Although the geometry and radius of the non-circular gear pair are different than the one defined in IVT, the ranges of the asymmetric pressure angles can be used to characterize the uncertainties of the pressure angles in this study. In the field of the reliability engineering, probabilistic physics-of-failure (PPoF) is commonly used in estimating probabilistic failure metrics such as failure rates of the targeted system [5]. The PPoF analysis includes two processes such as developing physics-of-failure model and uncertainty propagation. Lin et al. [3] and Li and Wu [4] 's study focused on the development of the physics-of-failure model that incorporated the underlying failure mechanisms with specific geometric dimensions of the non-circular gear pair. However, both lack the uncertainty quantification processes which is an important piece for PPoF methodology. Thus, this research makes

the following two contributions in the domain of the PPOF for non-circular gear pair system subjected to root bending fatigue:

- Develop a probabilistic-physics-of-failure methodology that accounts for the overloaded scenario and the high cycle fatigue for non-circular gear pair in IVT;
- Sensitivity analysis using different designs and material types for manufacturing the non-circular gear pair.

2. PROBABILISTIC PHYSICS-OF-FAILURE METHODOLOGY

As described, the PPOF methodology includes the physics-of-failure modelling and uncertainty propagation. Section 2.1 documents the physics-of-failure model development while Section 2.2 analyses the potential uncertainty included in this analysis.

2.1. Physics-of-Failure models

There are two potential underlying conditions causing the cyclic bending stress. The first condition arises from the time-dependent tidal current velocity that generates time varying power from the turbine in the tidal current energy converter. The turbine is connected to the IVT where non-circular gear pair is the first module in the IVT that takes the input power load. The second condition comes from the non-circular shape of the two gears, making the actual pitch-radius changes during the gear rotations. Therefore, in this paper, the first step is to build a model for stress evaluation from the two sources (i.e., time-dependent tidal current velocity and non-circular shape).

For the stress induced by the tidal current, the high-resolution model such as FEA would be an ideal choice; however, building an FEA model may require detailed geometrical information and a time-consuming effort before understanding the impacts of the physical parameters; therefore, in this paper, a theoretical equation for estimating the bending stress on the tooth is developed by combining the classical Lewis bending relation [6] with the case-specific pitch radius on the second non-circular gear [2] as shown in Equation (1).

$$\sigma_b = \frac{T_{in}(t)}{bmYr_{NG2}(t)}, \quad (1)$$

where σ_b is the bending stress that needed to be estimated as a function of the torque ($T_{in}(t)$) and the geometrical dismissions including face width (b), modules (m), Lewis form factor (Y), and pitch radius ($r_{NG2}(t)$). By applying Equation (1), it is assumed that the empirical Lewis form factor is applicable for the non-circular gear pair in IVT. The further validation and verification for the bending stress is required to be performed in FEA. The face width means the width of the gear tooth along the shaft axis while module indicates the size of each tooth. Lewis form factor is a function of the number of the tooth on the gear and the pressure angle formed by the tangential lines from the pitch circles of the two gears. While the real Lewis form factor should be also based on the real geometry of the non-circular gear pair, there are empirical tables developed by RoyMech [7] for spur gears that shows the ranges of the Lewis form factor. There are two pitch radiuses as shown in

Figure 1 where non-circular gear pair 1 (NG1) is the driven gear on the secondary shaft while non-circular gear pair 2 (NG2) is the driving gear on the input shaft. For this problem, the NG2 pitch radius is selected as the target gear and is a function of center distance (D) and the time varying ratio ($i_n(\theta(t))$) as shown in Equations (2) to (4) [2].

$$r_{NG2}(t) = \frac{D}{1+i_n(\theta(t))}, \quad (2)$$

$$i_n(\theta(t)) = \frac{\pi}{2\sqrt{2}} \max \left[\sin \theta_n, \sin \left(\theta_n - \frac{\pi}{2} \right), \sin(\theta_n - \pi), \sin \left(\theta_n - \frac{3\pi}{2} \right) \right], \quad (3)$$

$$\theta_n(t=0) = 0; \theta_n(t+1) = \theta_n(t) + \frac{2\pi N_p(t)}{60 i_n(\theta(t))}, \quad (4)$$

The input torque ($T_{in}(t)$) can be related to the input rotational speed ($N_p(t)$) on the non-circular gear pair in the unit of rotation per minute (rpm) and the transmitted power (P_t) from the turbine in the unit of W as shown in Equation (5).

$$T_{in}(t) = \frac{60P_t}{2\pi N_p(t)}, \quad (5)$$

The input rotational speed is obtained from the laboratory water testing data from [1]. The parameters used in physics-of-failure model in this research is summarized in Table 1.

Table 1: Parameters used in the Physics-of-Failure Models.

Parameters	Notations	Values	Units	References
Face width	b	0.02	m	[2]
Modules	m	0.003	m	[2]
Lewis form factor	Y	0.394*	Unitless	[7]
Center distance	D	0.135	m	[2]
Transmitted power from the turbine	P_t	750	W	[1]
Pitch radius for NG2-Case 1	$r_{NG2}(\varphi(t))$	0.064 to 0.076	m	[2]
Pitch radius for NG2-Case 2	$r_{NG2}(\varphi(t))$	0.064 to 0.076	m	[2]
Pitch radius for NG2-Case 3	$r_{NG2}(\varphi(t))$	0.064 to 0.076	m	[2]
Input rotational speed (150-200 rpm)-Case 1	$N_{p,1}(t)$	142 to 214	rpm	[1]
Input rotational speed (150-175 rpm)-Case 2	$N_{p,2}(t)$	137 to 186	rpm	[1]
Input rotational speed (175-200 rpm)-Case 3	$N_{p,3}(t)$	164 to 225	rpm	[1]

2.2 Uncertainty characterization and propagation

As shown in Table 1, there are two types of the parameters subjected to uncertainties as described previous that contributes the variation of stress. These are the sources of the aleatory uncertainty

* Assume that it is applicable for 42-tooth with 20 degrees of pressure angle

originating from the inherent variability from the tidal current. There should be an epistemic uncertainty associated with the measurement error when dealing with the real tidal current which is out of the scope of the analysis. Based on the available data, the input rotational speed for the three different cases were represented as the sinusoidal function as shown in **Figure 2** while the pitch radius for the second non-circular gear follows the co-sinusoidal function as shown in **Figure 3**. In addition, the pressure angles also have uncertainties if the IVT is in the process of the design and the optimized solution has not been provided. The uncertainty associated with the pressure angle is the source of epistemic uncertainty originating from the lack of knowledge about this real number. Without losing generality and ensuring the positive values for the pressure angle, it is assumed that a uniform distribution is appropriate to characterize the epistemic uncertainty associated with the pressure angle with mean value of 20 degree while the upper bound and lower bound of 30 and 10 degrees respectively [4]. For demonstration purpose, the uncertainty associated with the pressure angle is represented by sensitivity study.

Figure 2. Time-dependent plots for input rotational speed with three cases.

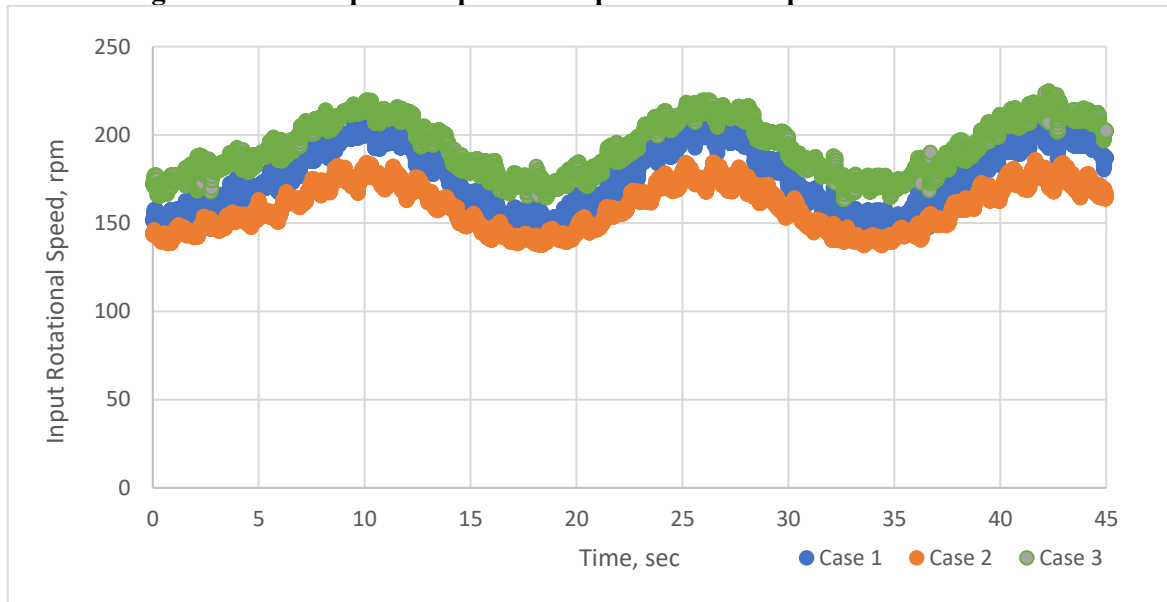
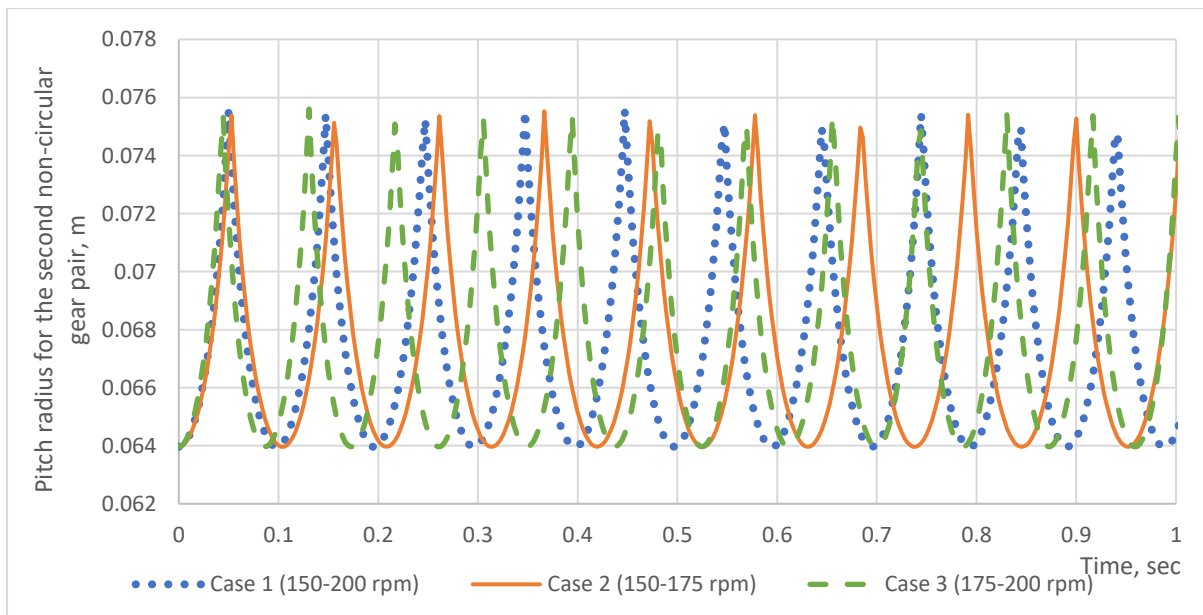


Figure 3. Time-dependent plots for pitch radius (r_{NG2}) with three cases.



2.3. Reliability Analysis for IVT

In the author's previous research, Equation (6) shows a simple reliability model to characterize the reliability of the entire IVT considering all the modules are in series.

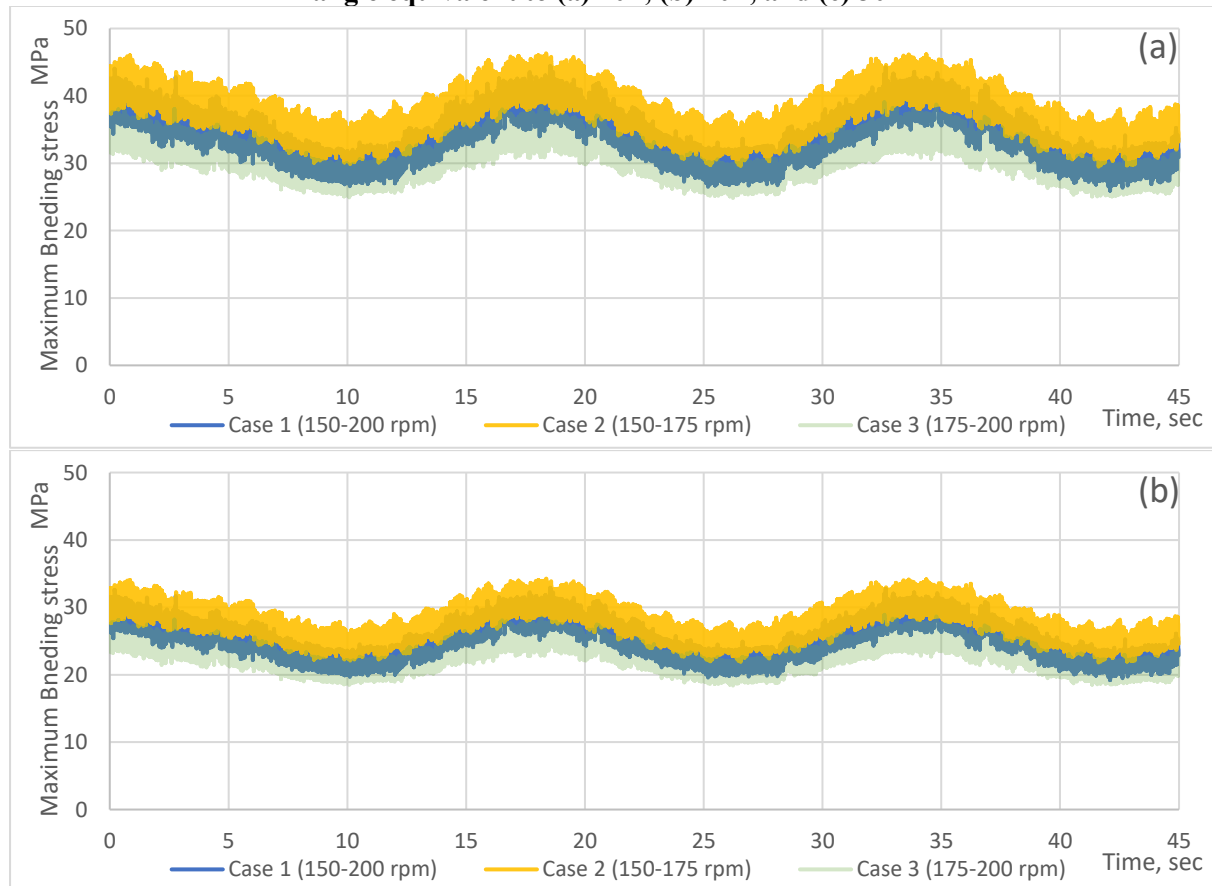
$$Re_{IVT} = Re_{NGP} * Re_{ICM} * Re_{MCM}, \quad (6)$$

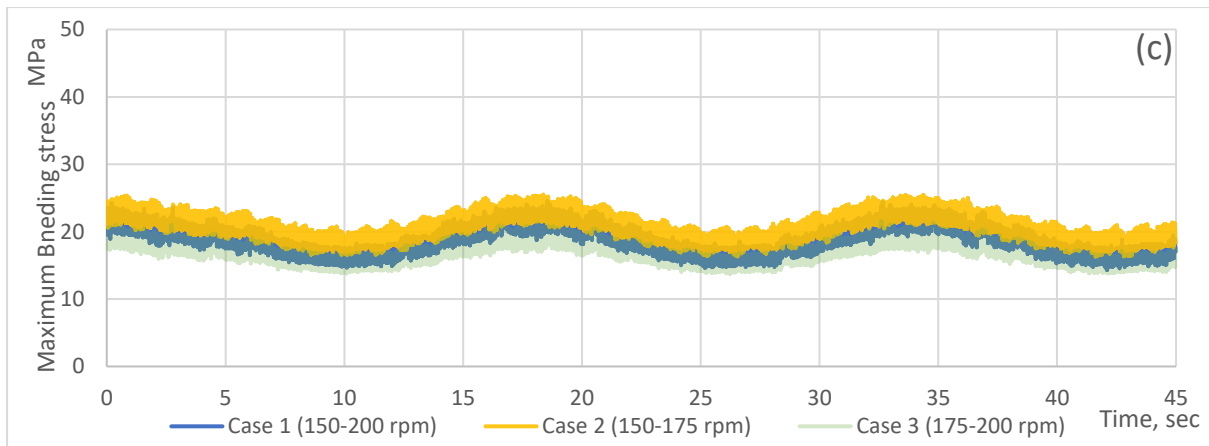
where Re_{NGP} is the reliability of the non-circular gear pair. Re_{ICM} is the reliability of the Input-Control Modul while the Re_{MCM} is the reliability of the motion-conversion module. This study focuses on the reliability estimation for non-circular gear pairs. Since limited reliability testing data is available, this research utilized the PPoF-based approach to estimate the probability of failure. The failure process should be modeled using two phases. The first phase evaluates if the maximum bending stress exceeds the material yield strength using stress-strength analysis (Section 2.3.1). This analysis tries to estimate the probability of sudden breaks and overloaded conditions. The second phase is to evaluate the total expected lifetime for the non-circular gear pair under cyclic stress conditions inducing the tooth bending fatigue. This phase targets on when the developed system expects to fail, which is an important piece in the reliability assessment. The total reliability of the entire non-circular gear pairs should integrate the results of the first two phases.

2.3.1. Phase I: Instantaneous overloaded analysis using stress- strength method

As shown above, the input rotational speed and the pitch radius change as a function of time, leading to stress variation on the second gear pair as shown in Figure 4 with different pressure angles.

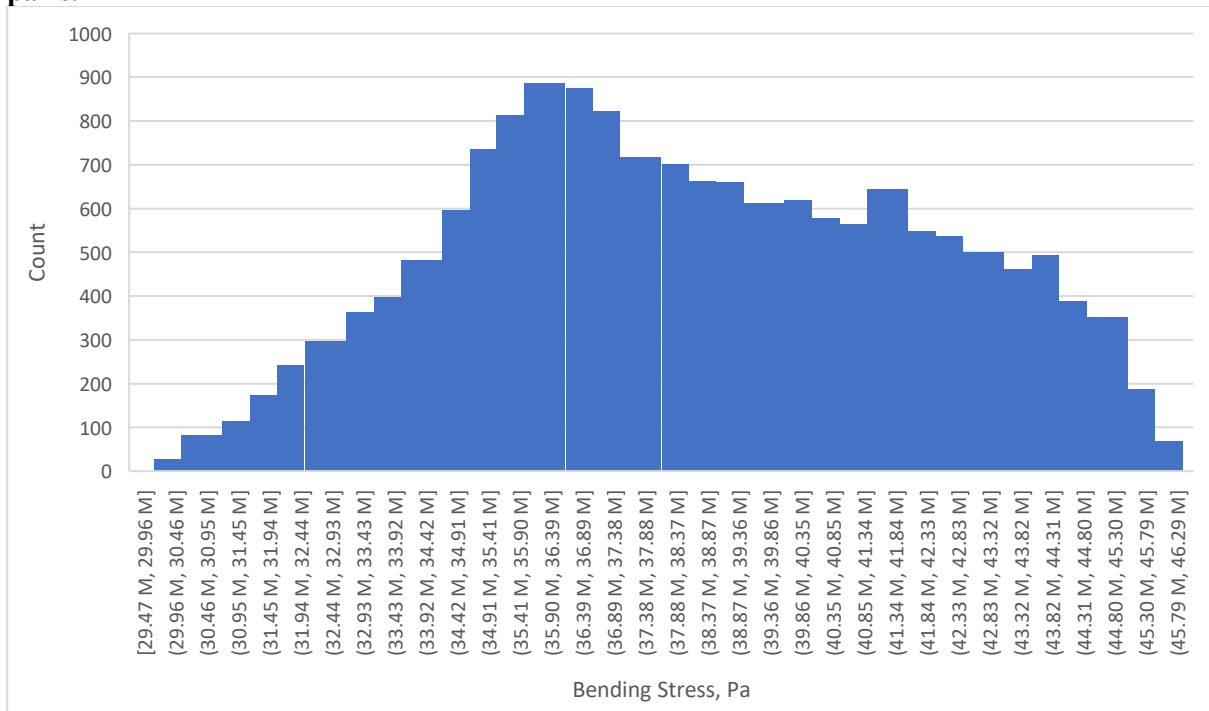
Figure 4. Time-dependent plots for maximum bending stress with three cases with pressure angle equivalent to (a) 10 °, (b) 20 °, and (c) 30 °.





From **Figure 4**, the sensitivity of the pressure angle can be shown but generally the bending stress is less than 50 MPa for all time. When looking at the cyclic stress, the stress ranges between the maximum and minimum stresses are less than 17 MPa as indicated in **Figure 4**. This small cyclic stress is induced by the small variations of the input rotational speed used in the laboratory testing. The real tidal speed can be used later once the TCECs are designed and tested in the tidal environment. As a bounding analysis, Case 2 in **Figure 4** (a) is selected for the stress-strength analysis and the Figure 5 shows the histogram of stress distribution.

Figure 5. Histogram of the stress distribution for Case 2 in Figure 4 (a) of the non-circular gear pairs.



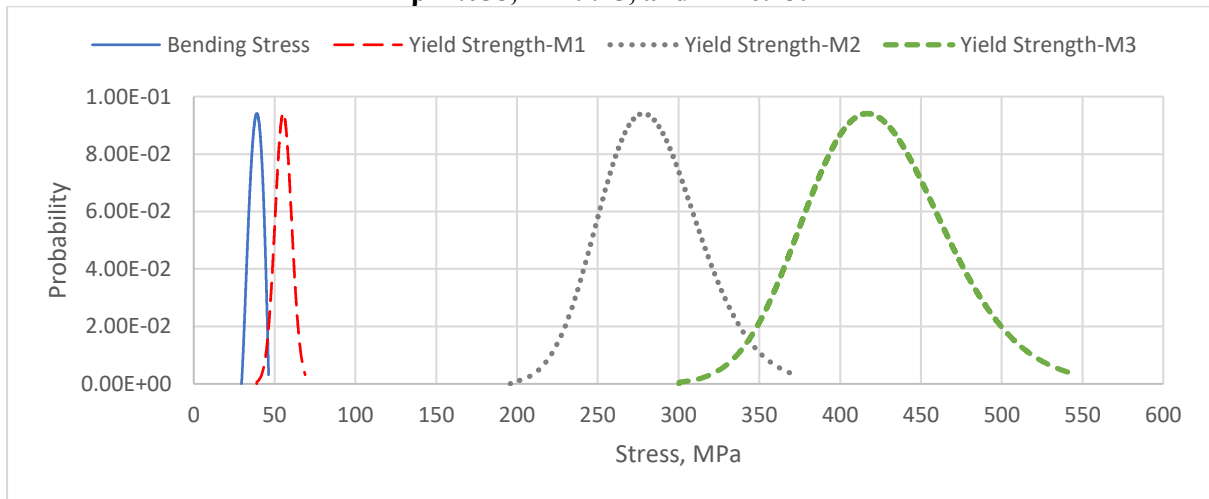
The strength of the material is defined as the yielding point beyond which the material transitions from elastic deformation to plastic deformation. This is a conservative assumption since for most of the ductile materials used in the IVT equipment, there should be time for plastic deformation before it goes to rupture or structural failure. Depending on the selection of the material for the non-circular gear pair, the strength of the material can vary. For the non-circular gear pair, it can be produced by the three common types of material as shown in **Table 2**. Usually, the higher strength indicates the higher cost of the materials.

Table 2: Strength for the selected materials to be used in the non-circular gear tooth.

Types of Material	Yield strength (MPa)	References
M1: Nylon-12	Normal ($\mu=55, \sigma=4$)	[8]
M2: 6061-T6/T651 aluminum	Lognormal (mean=276, std=23.6)	[9]
M3: AISI 4140 low-alloy steel	Lognormal (mean=415, std=33)	[10]

Based on stress and strength characterization, the instantaneous failure probability can be estimated using the stress-strength analysis when the bending stress is higher than the yield strength. For this specific case, the bending stress will be always less than the strength for material 6061-T6/T651 and AISI 4140. However, for the Nylon-12 material, it is possible that stress can exceed the strength as shown in **Figure 6**, where the overlapped area represents the probability of failure in phase I is around $4.71\text{E-}3$, leading to a reliability of 99.53% if only stress-strength analysis is considered. Repeating this process, the reliability for pressure angle of 20 and 30 degrees are

Figure 6. Stress-Strength analysis for non-circular gear pairs for three different types of material. The bending stress distribution was fitted to the beta distribution with $\alpha=2.37$, $\beta=2.058$, $A=29.45$, and $B=46.43$.



2.3.1. Phase II: Tooth bending high-cycle fatigue using S-N curves

Under conditions where stress is low, it can still cause failure under the cyclic condition such as the tooth bending fatigue. The expected lifetime of the non-circular gear pair can be estimated based on the S-N curve for specific material. For Case 2 with pressure angle of 10, 20 and 30 degrees, the maximum and minimum stress can be estimated to calculate the stress range, which serves as the foundation to calculate the numbers of cycles to failure and the corresponding time to failure using the Equation (7) based on [11].

$$t_{life} = \left(10^{-\frac{\log[(\sigma_{max}-\sigma_{min})^{297.75}]}{0.274}} \right) t_f / 86400, \quad (7)$$

where the parameters are shown in Table 3.

Table 3: Lifetime estimation for Case 2 with various pressure angles for M1 material.

Pressure Angle	10 degrees	20 degrees	30 degrees
Maximum stress: σ_{max} , MPa	46.3	34.3	25.5
Minimum stress: σ_{min} , MPa	29.5	21.8	16.3
Period of the cycle: t_f , sec	17.5	17.5	17.5
Lifetime: t_{life} , year	7.2	21.5	62.7
Failure rate, λ (1/year)	0.14	0.05	0.02
Reliability (one year),	86.94%	95.12%	98.02%

$Re_I(t=1) = e^{-\lambda t}$			
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Using the estimated failure rate based on the expected lifetime estimation, the reliability of the component can be estimated by assuming a constant failure rate one year using exponential distributions as shown in Table 3.

3. COMBINED RELIABILITY RESULTS

The total reliability of the non-circular gear pair can be constructed using the fault tree where the non-circular gear can fail either by overloaded or the tooth bending fatigue. Based on the fault tree, the reliability of the combined phases is estimated in Equation (8), where F_{PhaseI} and $F_{PhaseII}$ represents the failure probability in Phase I and Phase II, respectively.

$$Re_{NGP} = 1 - (F_{PhaseI} + F_{PhaseII} - F_{PhaseI} * F_{PhaseII}), \quad (8)$$

Table 4: Reliability for non-circular gear pair for various pressure angels using M1 material

Pressure Angle	10 degrees	20 degrees	30 degrees
F_{PhaseI}	4.71E-3	<1.0E-6	<1.0E-6
$F_{PhaseII}$	1.31E-1	4.88E-2	1.98E-2
Re_{NGP}	86.49%	95.12%	98.02%

Comparing Table 3 and Table 4, the reliability for Phase I does not have significant impact on the 20 degrees and 30 degrees scenario and the phase II results are dominant.

4. CONCLUSION

The infinitely variable transmission is newly developed equipment that can be used in the tidal current energy converters. The FMEA results indicate that the root bending fatigue is one of the dominant failure mechanisms in IVT. It is critical to analyze the reliability of the entire IVT and especially the reliability of the non-circular gear pairs due to the direct power input flow and the specific geometry where no reliability data is available. Two phases of probabilistic physics-of-failure method including the overloaded analysis and tooth-bending high cycle fatigue analysis are analyzed to find the combined reliability of the non-circular gear pairs. Reliability ranges from 86% to 98% depending on the design pressure angle if a Nylon-12 is used. Higher reliability can be expected if the other stronger materials like 6061-T6/T651 or AISI 4140 low-alloy steel are used. The future research should target real tidal current applications and with higher transmitted power to evaluate the reliability of the entire IVT.

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